With the growing market competition in the modern days, it has become essential for companies to keep a track on their product quality, and control it throughout. This helps firms and organizations to maintain a good reputation in the headlines and in the conscious of their consumers. All industries, be it pharmaceuticals, food production, or manufacturing use Likelihood Ratio Test as a statistical tool for product quality control to compare two or more manufacturing processes and determine whether they produce products of similar quality.

The LRT involves estimating the likelihood of data under two hypothesis: the null hypothesis that the mean quality of the two products is the same, and alternative hypothesis that the mean quality of the new product is greater than that of the original product. To perform the said test, companies might randomly sample their products to test them for quality to calculate a likelihood ratio statistic and test it against a pre-determined threshold.

Quality control is a critical process in manufacturing that ensures products meet certain standards and specifications. One company that has a well-established quality control system is Toyota. Toyota's production system, known as the Toyota Production System (TPS), is based on the principles of just-in-time manufacturing and continuous improvement. TPS includes several quality control tools and techniques, including the use of statistical process control (SPC) and the LR test.

SPC is a quality control method that uses statistical methods to monitor and control a production process. It involves collecting and analysing data on product quality during production to detect and correct any defects or variations. The LR test is used in SPC to test for the significance of changes in the production process, such as changes in machine settings or materials used.

A real-life situation where quality control is critical is the recall of millions of Toyota vehicles in 2009 and 2010 due to issues with unintended acceleration. One of the factors that contributed to the issue is a change in the design of the accelerator pedal assembly that affected its performance. Toyota used SPC and the LR test to identify the cause of the issue and implement corrective actions.

To illustrate the use of the LR test in quality control, consider a company which produces switches, and they want to know if a change in their manufacturing process brings about a major difference in the quality of the product. The company will start by choosing a parameter to compare the two batches of the product, let’s say tensile strength. The next step would be forming the hypothesis. The null hypothesis will be that both the processes produce switches with same tensile strength, while the alternative will be that the tensile strength of products in the two batches is different.

Let’s say this company takes a sample 20 units of switches from original and new batch each. On conducting studies, it is found that average tensile strength of the new product batch is 55 N/mm2 with a variance of 25 N/mm2. On the other hand, the original product batch has an average tensile strength of 50 N/mm2, with a variance of 25 N/mm2. Let’s consider the tensile strength of the switches in both batches to be normally distributed.

To calculate the likelihood of the null hypothesis, we assume that the two batches have the same mean and estimate a common mean by using the concept of weighted mean which comes out to be 52.5 N/mm2. The likelihood of null hypothesis comes out to be 2.57 \* 10-19. On the other hand, the likelihood of alternate hypothesis comes out to be 1.38 \* 10-19. When put into the formula for likelihood ratio, these values give us 2.04.

Now, to have a threshold value, let’s consider the level of significance to be 0.05. The critical value of chi-square distribution with one degree of freedom is 3.84. Since, our LR value is 2.04, which is less than the critical value, we fail to reject the null hypothesis. This means that the company can safely conclude that mean tensile strength of the new product is more or less the same with the previous product.

In conclusion, quality control is a critical process in manufacturing that ensures products meet certain standards and specifications. Companies like Toyota use statistical process control and the LR test to monitor and control the production process and identify any issues that may arise. The use of these tools helps companies maintain consistent product quality and reduce the likelihood of product recalls or other quality-related issues.

Credit risk management is crucial for financial institutions to assess and mitigate the potential risks of loan defaults, bankruptcies, and other credit-related losses. Effective credit risk management helps institutions to maintain a healthy portfolio, reduce credit losses, and ensure long-term profitability. Without proper credit risk management, financial institutions may face significant financial losses and reputation damage.

One company that uses the LR test in credit risk management is American Express. American Express has a sophisticated credit risk management system that uses various statistical models, including the LR test, to evaluate the creditworthiness of customers and make lending decisions. The company uses data analytics to analyze customers' payment behavior, credit history, income, and other factors to assess the likelihood of default. Based on this analysis, the company sets credit limits and interest rates for its customers.

Model Selection:

American Express uses the likelihood ratio test to evaluate the power of two different models for assessing credit risk.

American Express may have two different models for predicting credit risk or worthiness, a simple model with fewer parameters and a complex model with more parameters:

Model A: Parameters: credit utilization, payment history, and length of credit history (traditional credit scoring factors)

Model B: traditional credit scoring factors + income, employment history, and education level.

To determine which model is a better fit for assessing credit risk, American Express used the likelihood ratio test. The test involved comparing the likelihood of observing the credit data given Model A and the likelihood of observing the same data given Model B. It was found that Model B provides a significantly better fit to the data, then American Express concluded that Model B is a more reliable and accurate model for assessing credit risk.

Hypothesis Selection:

In this study, American Express was looking to improve its credit risk management practices. It wants to determine if there is a significant difference in default rates between two groups of borrowers: those with good credit scores and those with poor credit scores.

To perform this analysis, American Express decides to use a likelihood ratio test.

The null hypothesis is that there is no significant difference in default rates between the two groups, while the alternative hypothesis is that there is a significant difference.

It calculates the likelihood of the observed data given the null hypothesis and the alternative hypothesis. Using the likelihood ratio test, the bank compares the ratio of the two likelihoods to a threshold value to determine if the difference in default rates is statistically significant or not.

American Express found that the likelihood ratio is above the threshold, indicating that the difference in default rates between the two groups is statistically significant and not just due to chance. Based on this analysis, the bank decided to adjust its credit risk management practices, such as increasing the interest rates or tightening the lending criteria for borrowers with poor credit scores to reduce the risk of default.

The final inference shows that the likelihood ratio test is a useful tool for comparing the relative merits of different models and can provide valuable insights into the accuracy and reliability of credit risk assessments. The LRT helps in identifying significant differences in default rates between different groups of borrowers and in determining the most accurate model for assessing credit risk. By adopting effective credit risk management practices based on the LRT, financial institutions can reduce potential losses and maintain a healthy loan portfolio.